

SMU-HEP-93-24
hep-ph/9311377
November 1993

Singular defects with fractional topological charge in σ models and gauge theories.¹

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Abstract

A novel class of self-dual solutions in σ models and gauge theories is considered. The contribution of the corresponding fluctuations to the chiral condensate is calculated. We discuss the few tightly connected problems, such as the $U(1)$ problem, the θ dependence and the chiral symmetry breaking within a framework of this approach. Arguments in favour of significance of the configurations with fractional topological charge are given.

¹*Contribution to the Proceedings of Lattice-93, Dallas, 12-16 October, 1993;*

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1 Motivation. Why fractional topological charge should be considered?

Let me start these notes from the well known results concerning the chiral condensate in supersymmetric models. More specifically, let us consider 2 dimensional SUSY CP^{N-1} model, [1]. As is known the model possesses naive $U(1)$ chiral symmetry, which is broken by anomaly. However, the discrete symmetry $\sim Z_N$ is conserved. At large N this model can be explicitly solved and shows up the nonzero value for $\langle \bar{\psi}\psi \rangle$, which corresponds to discrete symmetry breaking phenomenon.

At the same time, the instanton can ensure non-zero value for the correlator $\langle \prod_{i=1}^N \bar{\psi}\psi(x_i) \rangle$ only, and not for the condensate $\langle \bar{\psi}\psi \rangle$. The reason is trivial and related to the fact, that one instanton transition is always accompanied by the emission of $2N$ fermionic zero modes. By *clustering*, at $(x_i - x_j) \rightarrow \infty$ this relation implies a nonvanishing magnitude for condensate as well, however $\langle \bar{\psi}\psi \rangle_{inst} = 0$ because we have $2N$ and not 2 zero modes. Besides that, in according with Witten index [2] we have N different vacuum states classifying by the phase of condensate $\langle \bar{\psi}\psi \rangle \sim \exp(\frac{2\pi i k + \theta}{N})$. Let us note that θ dependence comes through $\frac{\theta}{N}$. Such a function can be periodic in θ with period 2π only if there are many (N) vacuum states for given values of θ . I have to note that the same situation takes place in the supersymmetric gluodynamics as well as in SQCD. I refer to the review paper [3] on this subject in supersymmetric four dimensional models, but here I want to make a remark that such behavior is not specific for SUSY models.

In particular, the analogous θ/N dependence was discovered in gluodynamics at large N [4],[5], [6]. In these papers was argued that the vacuum energy at large N appears in the form $E \sim E(\theta/N)$. This fact actually is coded in the effective lagrangian containing the multi-branched logarithm $\log \det(U)$. In the Veneziano approach [5] the same fact can be seen from the formula for multiple derivation of the topological density Q with respect to θ at $\theta = 0$.

$$\frac{\partial^{2n-1}}{\partial \theta^{2n-1}} \langle Q(x) \rangle \sim \left(\frac{1}{N}\right)^{2n-1}, n = 1, 2, \dots \quad (1)$$

It is clear that it corresponds to the following θ dependence of the topological density $\langle Q \rangle \sim \sin \frac{\theta}{N}$. Let me remind, that the θ is the physical parameter of the theory and the θ - dependence of physics is linked to the $U(1)$ problem

[4],[5]. Indeed, if we believe that the resolution of the $U(1)$ problem appears within the framework of these papers, we must assume that the correlator

$$K = i \int d^4x \langle 0 | T Q(x), Q(0) | 0 \rangle \quad (2)$$

is nonzero in pure gluodynamics. But the topological susceptibility K is nothing but divergence of $\langle Q \rangle$ with respect to θ . As is known $K \sim \frac{1}{N}$. It demonstrates one more times that θ parameters comes to the theory through θ/N .

The question we want to raise can be formulated as follows. How can one reproduce θ/N dependence in the theory with integer topological charges only? Our answer is: *The configurations with fractional topological charges with finite action should be introduced to the theory.*

2 Basic assumptions

- I extend the class of admissible gauge transformation in gluodynamics. Thus, I allow the configurations with fractional topological charge (one half for $SU(2)$ group) in the definition of the functional integral. I call this configurations *toron*³. This extension means that a multivalued functions will appear in the functional integral. However, the main physical requirement is: *all gauge invariant values must be singlevalued*. Thus, the different cuts accompany the multivalued functions should be unobservable. Let us note that at large distances the *toron looks like a singular gauge transformation*.

- The next main point of the toron approach may be formulated as follows. We hope that in the functional integral of the gluodynamics, only certain field configurations (the toron of all types) are important. In this case, the consistency of these assumptions can be checked by considering a few simple models, where, on the one hand, the results are well known beforehand and, on the other hand they can be reproduced by the toron calculations [8].

³ We keep the term "toron", introduced in ref.[7]. By this means we emphasize the fact that the considering solution minimizes the action and carries the topological charge $Q = 1/2$, i.e. it possesses all the characteristics ascribed to the standard toron [7].

3 $0(3)\sigma$ model. Lessons and Experience.

We define the action and the topological charge of the supersymmetric $0(3)\sigma$ model, equivalent to the CP^1 theory as follows [1]:

$$S = \frac{1}{f} \int d^2x |D_\mu n|^2, \quad D_\mu = \partial_\mu - iA_\mu, \quad (3)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad Q = \frac{1}{4\pi} \int d^2x \epsilon_{\mu\nu} F_{\mu\nu}, \quad (4)$$

Here n_α is a complex 2-component unit spinor, transforming according to the fundamental representation of $SU(2)$, $A_\mu = -i\bar{n}\partial_\mu n$ is an auxiliary gauge field, and we have shown only bosonic part of the action. The classical solution in this language is determined by analytical function $P_\alpha(z)$, where $z = x_1 + ix_2$ and $n_\alpha = P_\alpha/|P|$. In particular, the standard instanton solution takes the form:

$$n_{inst} = \frac{1}{\sqrt{|z - z_0|^2 + \rho^2}} (z - z_0)_{z \rightarrow \infty} \Rightarrow e^{i\phi} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (5)$$

and becoming a pure gauge transformation at large distances. We wish to describe the solution n_t which at large distances looks as a pure gauge field $n_t(z \rightarrow \infty) \Rightarrow e^{i\phi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ with one half phase $\phi/2$ instead of integer phase ϕ (5) in order to describe one half topological charge. Besides that, we would like to regularize this behavior at small distances by parameter $\Delta \rightarrow 0$ in the very special way in order to preserve the selfduality equation:

$$n_t = \frac{1}{\sqrt{|z - z_0| + |\Delta|}} \left(\frac{\sqrt{|\Delta|}}{\sqrt{z - z_0}} \right)_{z \rightarrow \infty} \Rightarrow e^{i\phi/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (6)$$

As was expected, this solution is a double-valued function and it is defined on a covering space. Main physical requirement to these configurations is: all *gauge invariant values should be singlevalued*. In particular, the classical action is $S_t = 1/2 S_{inst}$ and the corresponding density is singlevalued and in the limit $\Delta \rightarrow 0$ goes to $\delta^2(x - x_0)$ function:

$$S = \lim_{\Delta \rightarrow 0} \frac{\Delta}{2f} \int \frac{d^2x}{|x|(|x| + |\Delta|)^2} \rightarrow \frac{\pi}{f} \int d^2x \delta^2(x) \quad (7)$$

With regularized expression (6) it can be easily checked that the number of fermionic (and bozonic) zero modes (ZM) equals two and not four, as in instanton case. The toron measure in the supersymmetric version of the model is given by

$$Z_t \sim M_0^2 d^2 z_0 \frac{d^2 \epsilon}{M_0} e^{\frac{-\pi}{\mathcal{F}}} = m d^2 z_0 d^2 \epsilon, \quad m = M_0 e^{\frac{-\pi}{\mathcal{F}(M_0)}} \quad (8)$$

where m is renormalization invariant combination. Now we are ready to calculate the chiral condensate in the model. Substituting the ZM in place of ψ , and recalling the integration over the collective variables satisfies $\int \epsilon^2 d^2 \epsilon = 1$, we verify that

$$\langle Q_5 = 2 | \bar{\psi}_L \psi_R | Q_5 = 0 \rangle \sim \int d^2 z_0 \bar{\psi}_0 \psi_0 = m \quad (9)$$

Because the transition amplitude (9) is non-zero, and because the toron transition changes the chiral charge Q_5 by two units, the true physical states $|\Omega_{\pm}\rangle$ must be superposition of the states $|Q_5 = 0, 2\rangle$. These two vacuum states are true physical vacua of spontaneously broken discrete chiral symmetry:

$$\langle \Omega_{\pm} | \bar{\psi}_L \psi_R | \Omega_{\pm} \rangle = \pm m \quad (10)$$

Besides that, it can be explicitly checked that these physical vacua provide the correct $\theta/2$ dependence and thus, the θ evolution from $\theta = 0$ to $\theta = 2\pi$ renumbers two degenerate states Ω_{\pm} . Let me repeat, that as soon as we allowed one half topological charge, *the number of the classical vacuum states is increased* by the same factor two in comparison with the standard classification, counting only integer winding numbers $|n\rangle$. This result is in a full agreement with large N results presented above. Analogous calculations can be done in four-dimensional case and we refer to the original papers [8].

4 Conclusion, Interpretation, Problems.

- We interpret the standard instanton as the pseudoparticle, constructed from these singular points defects. In particular, for 2-dimensional CP^{N-1} model we interpret $2N$ boson ZM accompanied by instanton as translation modes for N different torons. Four dimensional instanton for any gauge

group G can be interpreted in the same way. In this case as is known the number of bosonic ZM equals $4C(G)$, where $C(G)$ is Casimir operator ($C(SU(N)) = N$). This number we interpret as translations of $C(G)$ torons. This conjecture, in particular, is in agreement with formula for the instanton measure in supersymmetric gluodynamics

$$Z_{inst} \sim \prod_{i=1}^{C(G)} d^4 x_i d^2 \epsilon_i (e^{-\frac{8\pi^2}{g^2 C(G)}})^{C(G)} \sim \prod_i^{C(G)} Z_t(i), \quad (11)$$

ensures the correct renormalization invariant dependence and gives the correct dependence on θ for gluino condensate.

- The direct consequence of our definition of the functional integral is the appearing of the new quantum number classifying the vacuum states. Indeed, as soon as we allowed one half topological charge, the number of the classical vacuum states is increased by the same factor two in comparison with a standard classification, counting only integer winding numbers $|n\rangle$. This is exactly what we observed from the large N analysis.

Of course, vacuum transitions eliminate this degeneracy. However the trace of enlargement number of the classical vacuum states does not disappear. Vacuum states now classified by two numbers : $0 \leq \theta < 2\pi$ and $k = 0, 1$. These is in agreement with large N results where the nontrivial θ dependence in pure YM theory comes through θ/N [6] at large N and we had N additional states for each given θ .

- I would like to stress that a lot of problems (like the θ dependence, the $U(1)$ problem, the counting of the discrete number of vacuum states, the nonzero value of the vacuum energy and so on...) can be described in a very simple manner from this uniform point of view.

- Main question to Lattice Community: How can one describe these singular self-dual (with finite action) defects on the Lattice? Some results on this subject have been presented by Antonio Gonzalez-Arroyo to this Proceedings. This work is supported by the TNRLC grant No 528428

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